

Forecasting the South African Economy: A DSGE-VAR Approach

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Abstract

This paper develops an estimated hybrid model that combines the micro-founded DSGE model with the flexibility of the theoretical VAR model. The model is estimated via the maximum likelihood technique based on quarterly data on real Gross National Product (GNP), consumption, investment and hours worked, for the South African economy, over the period of 1970:1 to 2000:4. Based on a recursive estimation using the Kalman filter algorithm, the out-of-sample forecasts from the hybrid model are then compared with the forecasts generated from the Classical and Bayesian variants of the VAR for the period 2001:1-2005:4. The results indicate that, in general, the estimated hybrid DSGE model outperforms the Classical VAR, but not the Bayesian VARs in terms of out-of-sample forecasting performances.

Journal of Economic Literature Classification: E17, E27, E32, E37, E47.

Keywords: DSGE Model; VAR and BVAR Model; Forecast Accuracy; DSGE Forecasts; VAR Forecasts; BVAR Forecasts.

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1 Introduction

The controversy about the methods for evaluating the empirical relevance of economic models is not new. However, two distinct approaches has emerged since the early 1980s. First, the standard econometric approach in which an economic model should be embedded within a complete probability model and analyzed using statistical methods (Watson, 1993). For instance, the vector autoregressive (VAR) models introduced by Sims (1980), which can be taken directly to the data to perform statistical hypothesis. The VAR models also became popular in the forecasting literature pioneered by Litterman (1986b). Although the VAR models have been proved to be reliable tools in terms of data description and forecasting, but they are subject to Lucas critique (Lucas, 1976) and also fail to take account of the nonlinearities in the economy.

The second approach, pioneered by Kydland and Prescott (1982) and Long and Plosser (1983), has become increasingly popular for evaluating dynamic macroeconomic models. The dynamic stochastic general equilibrium (DSGE) models are explicitly derived from the first principles. DSGE models describe the general equilibrium of a model economy in which agents like consumers and firms maximize their objectives subject to their budget and resource constraints (Del Negro and Schorfheide, 2003). Therefore, the DSGE structural parameters, in principle, do not vary with policy. However, the calibrated DSGE models are too stylized to be taken directly to the data and often yield fragile results, at least at the first glance (Stock and Watson, 2001; Ireland, 2004).

Following Ireland (2004), we develop an estimated DSGE model for forecasting the Gross National Product (GNP), consumption, investment and hours worked for South African economy. The hybrid DSGE-VAR model combines the micro-founded DSGE model with the flexibility of the VAR model. The model is estimated via the maximum likelihood technique based on quarterly data obtained from the South African Reserve bank over the period of 1970:1 to 2000:4. Based on a recursive estimation using the Kalman filter algorithm, the out-of-sample forecasts from the hybrid model are then compared with the forecasts generated from the Classical and Bayesian variants of the VAR for the period 2001:1-2005:4.

This remainder of the paper is organized as follows. Besides the introduction, Section 2 lays out the theoretical model while Section 3 describes the hybrid model. Results are presented in Section 4 and Section 5 concludes.

2 The Model Economy

The model economy, here, is based on the benchmark real business cycle model developed by Hansen (1985). Equilibrium models have been criticized for depending heavily on individuals' substitution of leisure and work responding to the change in interest rate or wage. Hansen (1985) and Rogerson (1988) argue that in the real economy labor is indivisible. Individuals either work full time or not at all. Other features of Hansen's indivisible labor are exactly the same as standard real business model, such as Kydland and Prescott (1982). The economic environment is described below.

The model economy is populated by infinitely-lived households. The preferences of households are assumed to be identical. Households maximize the expected utility over life time:

$$U(C_t, H_t) = E_t \sum_{t=0}^{\infty} \beta^t (\ln C_t - \gamma H_t), \quad 0 < \beta < 1 \quad \gamma > 0 \quad (1)$$

where C_t and H_t are consumption and hours worked respectively, β is the discount factor that households apply to future consumption.

The technology is defined as a standard Cobb-Douglas production function with constant-returns-to-scale:

$$Y_t = Z_t K_{t-1}^{\rho} (\eta^t H_t)^{1-\rho}, \quad 0 < \rho < 1 \quad \eta > 1 \quad (2)$$

where ρ is the fraction of aggregate output that goes to the capital input and $1 - \rho$ is the fraction that goes to the labor input. η measures the gross rate of labor-augmenting technological process. Z_t is the technology shock, which is exogenously evolving according to the law of motion:

$$\log Z_t = (1 - \psi)\log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma^2) \quad (3)$$

where ψ and \bar{Z} are parameters, and $0 < \psi < 1$. The innovation ϵ_t is normally distributed.

As in a neoclassical growth model, capital stock depreciates at a constant rate of δ , and households invest a fraction of income in capital stock in each period. This amount of investment forms part of productive capital in current period. Therefore the law of motion for aggregate capital stock is

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad 0 < \delta < 1 \quad (4)$$

The model economy is a closed economy, where $Y_t = C_t + I_t$. In equilibrium the representative consumer maximizes his or her utility function (1) subject to the aggregate constraints

$$\begin{aligned} Y_t &= C_t + I_t \\ Y_t &= Z_t K_{t-1}^\rho (\eta^t N_t)^{1-\rho} \\ K_{t+1} &= (1 - \delta)K_t + I_t \\ \log Z_t &= (1 - \psi)\log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2) \end{aligned}$$

3 The Hybrid Model: A DSGE-VAR Approach

Kydland and Prescott (1982) argue that under the basic neoclassical framework, the U.S. business cycle fluctuations are purely driven by real technology shocks. This one-shock assumption makes real business cycle model stochastically singular. Using a version of the King *et al.* (1988) model, Ingram *et al.* (1994) point out that it is impossible to derive the realizations of the productivity shocks using a singular model if the variance-covariance matrix of the observable variables is actually nonsingular. In order to overcome this singularity problem, Ingram *et al.* (1994), DeJong *et al.* (2000a, b), Ireland (2001 and 2002), and Kim (2000) elaborate the DSGE model to a more elaborate model by including

as many shocks as there are endogenous variables in the model. This approach, in addition, can be served to identify sources of output variation¹.

Recently, Ingram and Whiteman (1994), DeJong *et al.* (2000a, b), and Schorfheide (2000) use the Bayesian framework to estimate and evaluate DSGE models. The principle of the Bayesian analysis of DSGE models is combining prior and likelihood functions to obtain posterior distributions of the functions of interest. However, different methods have been applied to this kind of research. Ingram and Whiteman (1994) use the King *et al.* (1988) real business cycle model as a source of priors in Bayesian VAR (BVAR) forecasting exercises, whereas, the method pursued by DeJong *et al.* (2000a, b) and Schorfheide (2000) lies between calibration and maximum likelihood estimation exclusively within the DSGE model. Moreover, there is a significant progress in the development of DSGE models that deliver acceptable forecasts (Smets and Wouters, 2003a, b, 2004; Del Negro and Schorfheide, 2004, Del Negro *et al.*, 2005). The authors use prior information derived from DSGE models in the estimation of a VARs. The hybrid models are then used to perform forecasting exercises. The empirical results suggest that the out-of-sample forecasts from the estimated DSGE models outperform the VARs estimated with simple least squares methods.

The approach proposed in this paper is based on Ireland (2004), which is different from the ones discussed above. We augment the linearized solution of the model with unobservable errors that have a VAR representation. This approach was developed originally by Sargent (1989) and pursued by Altug (1989), Watson (1993), Hall (1996), and McGrattan *et al.* (1997). The hybrid DSGE-VAR model is constructed as follows.

The approximated solution is applied to the log-linearized model², where a serially correlated residual is augmented to each equation as in (5)

$$\hat{\pi}_t = A\hat{x}_t + \mu_t \tag{5}$$

and

¹The literature suggest that the technology shocks are primarily responsible for the postwar U.S. business cycle fluctuations.

²Appendix B describes the steady state of the model as well as the the log-linearized model

$$\hat{x}_t = B\hat{x}_{t-1} + C\epsilon_t \quad (6)$$

$$\mu_t = D\mu_{t-1} + \xi_t \quad \xi_t \sim i.i.d.(0, \sigma_\xi^2) \quad (7)$$

where $\hat{\pi}_t$ is the vector of all de-trended endogenous variables in log-deviations, $\hat{\pi}_t = [\hat{y}_t \ \hat{c}_t \ \hat{i}_t \ \hat{h}_t]'$, and \hat{x}_t is the vector of de-trended state variables in log-deviations, $\hat{x}_t = [\hat{k}_t \ \hat{z}_t]'$. The matrix D is governing the persistence of the VAR residuals. The covariance matrix of the residuals in (7), $E\xi_t\xi_t' = V$, is uncorrelated with the innovation to technology, ϵ_t . The covariance matrix V is also constrained to be positive definite (Hamilton, 1994: 147).

Sargent (1989) assumes the measurement errors are uncorrelated with the data generated from the model by restricting D and V matrices as diagonal. In this paper, however, we estimate the DSGE model both with and without the restrictions on D and V matrices. The advantage of imposing no restrictions on D and V matrices is that the residuals in μ_t can capture not only the measurement errors, but also the movements and co-movements in the data that the stylized real business cycle model cannot explain (Ireland, 2004: 1210). Furthermore, in order to guarantee the residuals in μ_t are stationary, the eigenvalues of the matrix D , which govern the persistence of the VAR residuals, are constrained to be less than one.

The hybrid model is estimated based on quarterly data on real Gross National Product (GNP), consumption, investment and hours worked, for the South African economy, over the period of 1970:1 to 2000:4. The model economy is a closed economy (i.e. $Y_t = C_t + I_t$), where C_t and I_t are defined as final consumption expenditure by households and gross investment respectively³. The series are then converted into per-capita form by dividing them with the population aged between 15-64. Since there is no data for Hours worked, we generate the series follows. We assume employees work 40 hours per week and multiply it by the ratio of employment to labor force ratio.⁴

³Data are obtained from South African Reserve Bank Quarterly Bulletin, seasonally adjusted at constant price (2000 = 100).

⁴Data for employment is obtained from Statistics South Africa. Population aged 15-64 obtained from World Bank database is used as the proxy of labor force.

The hybrid model consisting of (5), (6), and (7) is in state-space form and can be estimated via a maximum likelihood approach. In our real business cycle model, output, consumption, and investment grow at the same rate of η in steady state. Before estimation, the series for output, consumption, and investment are de-trended by dividing with η . In addition, series for I_t is redundant in the estimation since the resource constraint holds by construction in the data. Therefore, $\hat{\pi}_t$, μ_t , and ξ_t is reduced to 3×1 vector:

$$\begin{aligned}\hat{\pi}_t &= [\hat{y}_t \ \hat{c}_t \ \hat{h}_t]' \\ \mu_t &= [\mu_{yt} \ \mu_{ct} \ \mu_{ht}]' \\ \xi_t &= [\xi_{yt} \ \xi_{ct} \ \xi_{ht}]'\end{aligned}$$

and for all $t = 1, 2, 3, \dots$, the matrices D and V are:

$$D = \begin{bmatrix} d_{yy} & d_{yc} & d_{yh} \\ d_{cy} & d_{cc} & d_{ch} \\ d_{hy} & d_{hc} & d_{hh} \end{bmatrix}; \quad V = \begin{bmatrix} v_y^2 & v_{yc} & v_{yh} \\ v_{cy} & v_c^2 & v_{ch} \\ v_{hy} & v_{hc} & v_h^2 \end{bmatrix}$$

The structural parameters, $\beta, \rho, \eta, \delta$, and ψ , are constrained to satisfy the theoretical restrictions discussed in Section 2. The discount factor β and capital depreciation rate δ are fixed in the estimation. The discount factor β is set equal to 0.99, as in Hansen (1985), which implies an annual real interest rate of four percent in steady state. The annual aggregate capital depreciation rate δ is obtained from annual averaged values of $\frac{I}{Y}$ and $\frac{K}{Y}$. This yields an annual depreciation rate of 0.076, or a quarterly rate of 0.019. The fixed β and δ together with the estimated ρ, η, γ , and z help match the steady state values of y, c, h in the model with those in the data, whereas ψ and σ only affect the model's dynamics.

4 Result

In this section, we compare the out-of-sample forecasting performance of the hybrid DSGE-VAR model with the VARs, both Classical and Bayesian, in terms of the Root Mean Squared Errors (RMSEs). Before this, however, it is important to lay out the basic structural differences and advantages of using BVARs over traditional VARs for forecasting.

4.1 Classical and Bayesian VARs

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$\chi_t = C + \lambda(L)\chi_t + \varepsilon_t \quad (8)$$

where χ is a $(n \times 1)$ vector of variables being forecasted; $\lambda(L)$ is a $(n \times n)$ polynomial matrix in the backshift operator L with lag length p , i.e., $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \dots + \lambda_p L^p$; C is a $(n \times 1)$ vector of constant terms; and ε is a $(n \times 1)$ vector of white-noise error terms. The VAR model, thus, posits a set of relationships between the past lagged values of all variables and the current value of each variable in the model.

A crucial drawback of the VAR forecasts is “overfitting” due to the inclusion too many lags and too many variables, some of which may be insignificant. The problem of “overfitting” results in multicollinearity and loss of degrees of freedom, leads to inefficient estimates and large out-of-sample forecasting errors. Thus, it can be argued the performance of VAR forecasts will deteriorate rapidly as the forecasting horizon becomes longer.

A forecaster can overcome this “overfitting” problem by using Bayesian techniques. The motivation for the Bayesian analysis is based on the knowledge that more recent values of a variable are more likely to contain useful information about its future movements than older values. From a Bayesian perspective, the exclusion restriction in the VAR is, on the other hand, an inclusion of a coefficient without a prior probability distribution (Litterman, 1986a).

The Bayesian model proposed by Litterman (1981), Doan, *et al.* (1984), and Litterman (1986b), imposes restrictions on those coefficients by assuming they are more likely to be near

zero. The restrictions are imposed by specifying normal prior⁵ distributions with zero means and small standard deviations for all the coefficients with standard deviation decreasing as lag increases. One exception is that the mean of the first own lag of a variable is set equal to unity to reflect the assumption that own lags account for most of the variation of the given variable. To illustrate the Bayesian technique, suppose the “Minnesota prior” means and variances take the following form:

$$\begin{aligned}\beta_i &\sim N(0, \sigma_{\beta_i}^2) \\ \beta_j &\sim N(0, \sigma_{\beta_j}^2)\end{aligned}\tag{9}$$

where β_i represents the coefficients associated with the lagged dependent variables in each equation of the VAR, while β_j represents coefficients other than β_i . The prior variances $\sigma_{\beta_i}^2$ and $\sigma_{\beta_j}^2$, specify the uncertainty of the prior means, $\beta_i = 1$ and $\beta_j = 0$, respectively.

Doan *et al.* (1984) propose a formula to generate standard deviations as a function of small number of *hyperparameters*⁶: w , d , and a weighting matrix $f(i, j)$. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few *hyperparameters*. The specification of standard deviation of the distribution of the prior imposed on variable j in equation i at lag m , for all i, j and m , defined as $S(i, j, m)$:

$$S(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_i}{\hat{\sigma}_j}\tag{10}$$

where:

$$f(i, j) = \begin{cases} 1 & \text{if } i = j \\ k_{ij} & \text{otherwise, } 0 \leq k_{ij} \leq 1 \end{cases}$$

⁵Note Litterman (1981) uses a diffuse prior for the constant, which is popularly referred to as the “Minnesota prior” due to its development at the University of Minnesota and the Federal Reserve bank at Minneapolis.

⁶The name of *hyperparameter* is to distinguish it from the estimated coefficients, the parameters of the model itself.

$$g(m) = m^{-d}, \quad d > 0$$

The term w is the measurement of standard deviation on the first own lag, which indicates the overall tightness. A decrease in the value of w results a tighter prior. The parameter $g(m)$ measures the tightness on lag m relative to lag 1, and is assumed to have a harmonic shape with a decay of d . An increasing in d , tightens the prior as lag increases.⁷ The parameter $f(i, j)$ represents the tightness of variable j in equation i relative to variable i . Reducing the interaction parameter k_{ij} tightens the prior. $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are the estimated standard errors of the univariate autoregression for variable i and j respectively. In the case of $i \neq j$, the standard deviations of the coefficients on lags are not scale invariant (Litterman, 1986b: 30). The ratio, $\frac{\hat{\sigma}_i}{\hat{\sigma}_j}$ in (31), scales the variables so as to account for differences in the units of magnitudes of the variables.

The BVAR model is estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. For each restriction imposed on the parameter estimated, the number of observations and degrees of freedom are increased by one in an artificial way. Therefore, the loss of degrees of freedom associated with the unrestricted VAR is not a concern in the BVAR.

4.2 Forecast accuracy

Table 1 to 4 report the root mean squared errors (RMSEs) from the hybrid DSGE-VAR model along with the VARs. In general, for all the four variables the hybrid model outperforms the unrestricted VAR.⁸ To be more precise, for output and consumption the unconstrained hybrid model does better than the constrained hybrid model and the unrestricted VAR. However, for hours worked the constrained hybrid model outperforms both the unconstrained

⁷In this paper, we set the overall tightness parameter (w) equal to 0.3, 0.2, and 0.1, and the harmonic lag decay parameter (d) equal to 0.5, 1, and 2. These parameters values are chosen so that they are consistent with the ones that used by Liu and Gupta (2007).

⁸The hybrid model has 21 parameters, the six structural parameters $\gamma, \rho, \eta, z, \psi$, and σ from the real business cycle model, the fifteen elements from matrix D and V governing the the behavior of the VAR residuals. For the constrained hybrid model, the number of parameters is reduced to 12. The VAR(1) model that we use to judge the hybrid model's out-of-sample forecasting performance also has 21 parameters, output, consumption, and hours worked together with a constant and a linear time trend.

one and the unrestricted VAR. The scenario for investment is a bit different. The unconstrained hybrid model does better than the constrained one for only the one-quarter and two-quarters ahead out-of-sample forecast, whereas for the three-quarters and four-quarters ahead forecast the constrained hybrid model outperforms the unconstrained one.

As far as the forecasting performances of the BVARs are concerned, it is clear that the BVARs improve the out-of-sample forecast performances significantly. The RMSEs⁹ generated from the BVARs are much smaller than those generated from both the hybrid model and the unrestricted VARs. In addition, the result suggests that a BVAR with a relatively loose prior produces smaller forecast errors. For all variables, output, consumption, investment and hours worked, a BVAR with the most loose prior ($w = 0.3$, $d = 0.5$) performs the best.

5 Conclusion

In this paper, we develop an estimated DSGE model in which we augment a vector of residuals that follows a VAR(1) process. Therefore the hybrid model combines the micro-founded DSGE model with the flexibility of the theoretical VAR model, hence the name of the model — DSGE-VAR. We then employ the hybrid model to measure the out-of-sample forecasting exercise for output, consumption, investment, and hours worked for the South African economy. The results indicate that, in general, the estimated hybrid DSGE model outperforms the Classical VAR but not the Bayesian VARs. Moreover, the results suggest that a BVAR with a relatively loose prior produces smaller out-of-sample forecast errors.

The Hansen’s (1985) version real business cycle model used in this paper is singular in the sense that the technology shock is the only shock to the system. Therefore, it is necessary to study the importance of various shocks in accounting for the dynamic behaviour of output and its main components. In this regard, future research aims to estimate a New Keynesian DSGE model which will allow us to incorporate nominal shocks. Further, we also aim to estimate the current model using Bayesian techniques. The ultimate goal of all these future

⁹Here we only report the BVAR with the prior that does the best in terms of the out-of-sample forecasting performance.

extensions will be analyzed whether the DSGE model can outperform the BVARs as far as forecasting is concerned.

References

- DeJong, D. N., Ingram, B. F. and C. H. Whiteman, 2000a. Keynesian impulses verses Solow residuals: identifying sources of business cycle fluctuations. *Journal of Applied Econometrics*, 15: 311-329.
- , 2000b. A Bayesian approach to dynamic macroeconomics. *Journal of Econometrics*, 98: 203-223.
- Del Negro, M. and F. Schorfheide, 2003. Take Your Model Bowling: Forecasting with General Equilibrium Models. Federal Reserve Bank of Atlantic, Economic Review, 4th Quarter 2003.
- , 2004. Priors from general equilibrium models for VARs. *International Economic Review*, 45(2): 643-673.
- Del Negro, M., Schorfheide, F, Smets, F. and R. Wouters, 2005. On the fit and forecasting performance of New-Keynesian models. European Central Banks, Working Paper, No. 491.
- Doan, T., Litterman, R. B. and C. Smis, 1984. Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews*, 3: 1-144.
- Hall, G. J., 1996. overtime, effort, and the propagation of business cycle shocks. *Journal of Monetary Economics*, 38: 139-160.
- Hansen, G. D., 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16: 309-327.
- Ingram, B. F., Kocherlakota, N. R. and N. E. Savin, 1994. Explaining business cycle: a multiple-shock approach. *Journal of Monetary Economics*, 34: 415-428.
- Ingram, B. F. and C. Whiteman, 1994. Supplanting the ‘Minnesota’ prior : forecasting macroeconomic time series using real business cycle model priors. *Journal of Monetary Economics*, 34(3): 497-510.

- Ireland, P. N., 2001. Money's role in the monetary business cycle. National Bureau of Economic Research, Working Paper 8115. Cambridge.
- , 2002. Endogenous money or sticky price? National Bureau of Economic Research, Working Paper 9390. Cambridge.
- , 2004. A method for taking models to the data. *Journal of Economics Dynamic & Control*, 28: 1205-1226.
- King R. G., Plosser, C. I. and S. T. Rebelo, 1988. Production, growth and business cycles: I. The basic neoclassical model. *Journal of Monetary Economics*, 21: 195-232.
- Kydland, F. E. and E. C. Prescott, 1982. Time to build and aggregate fluctuations. *Econometrica*, 50(6): 1345-1370.
- Litterman, R. B., 1981. A Bayesian procedure for forecasting with vector autoregressions. Working Paper, Federal Reserve Bank of Minneapolis.
- , 1986a. A statistical approach to economic forecasting. *Journal of Business and Statistics*, 4(1): 1-4.
- , 1986b. Forecasting with Bayesian vector autoregressions—five years of experience. *Journal of Business and Statistics*, 4(1): 25-38.
- Liu G. L. and R. Gupta, 2007. A small-skill DSGE model for forecasting the South Africa economy. *South Africa Journal of Journal of Economics*, 75: 2.
- Long, Jr. J. B. and C. Plosser, 1983. Real business cycles. *Journal of Political Economics*, 91: 39-69.
- Lucas, R. E. Jr., 1976. Econometric policy evaluation: A critique. Carnegie Rochester Conference Series on Public Policy, 1: 19-46.
- Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics*, 21: 3-16.

- McGrattan, E. R., Rogerson, R., and R. Wright, 1997. An equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review*, 38: 267-290.
- Sargent, T. J., 1989. Two models of measurements and the investment accelerator. *Journal of political Economy*, 97: 251-287.
- Schorfheide, F., 2000. Loss function-based evaluation of DSGE models. *Journal of Applied Econometrics*, 15: 645-670.
- Sims, C. A., 1980. Macroeconomics and reality. *Econometrica*, 48(January): 1-48.
- Smets, F. and R. Wouters, 2003a. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5): 1123-1175.
- , 2003b. Shocks and frictions in US business cycle: a Bayesian DSGE approach. Mimeo, European Central Banks.
- , 2004. Forecasting with a Bayesian DSGE model: an application to the Euro area. European Central Banks, Working Paper, No. 389.
- Stock, J. H. and M. W. Watson, 2001. Vector autoregressions. *Journal of Economic Perspectives*, 15(4): 101-105.
- Theil, H., 1971. Principles of econometrics. New York: John Wiley.
- Uhlig, H., 1995. A toolkit for analyzing nonlinear dynamic stochastic models easily. Discussion Paper 101. Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.
- Watson, M. W., 1993. Measures of fit for calibrated models. *Journal of Political Economy*, 101(6): 1011-1040.

A Optimization

In our model economy, the representative consumer problem is to maximize the utility function (1) by choosing $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$

$$U(C_t, H_t) = E_t \sum_{t=0}^{\infty} \beta^t (\ln C_t - \gamma H_t), \quad 0 < \beta < 1 \quad \gamma > 0$$

subject to the resource constraint:

$$Z_t K_t^\rho (\eta^t H_t)^{1-\rho} \geq C_t + K_{t+1} - (1 - \delta) K_t \quad (11)$$

From the (11), we have:

$$C_t = Z_t K_t^\rho (\eta^t H_t)^{1-\rho} + (1 - \delta) K_t - K_{t+1} \quad (12)$$

The Bellman equation for this problem:

$$V(K_t, Z_t) = \max_{H_t, K_{t+1}} \{ \ln [Z_t K_t^\rho (\eta^t H_t)^{1-\rho} + (1 - \delta) K_t - K_{t+1}] - \gamma H_t \} + \beta E_t V(K_{t+1}, Z_{t+1}) \quad (13)$$

Deriving the first order condition (FOC) for intratemporal condition:

$$\frac{\partial V(K_t, Z_t)}{\partial H_t} = \frac{1}{C_t} (1 - \rho) Z_t K_t^\rho \eta^{t(1-\rho)} H^{-\rho} - \gamma \quad (14)$$

$$\gamma = \frac{Y_t}{C_t} (1 - \rho) \frac{1}{H_t} \quad (15)$$

where the FOC for intertemporal condition:

$$\frac{\partial V(K_t, Z_t)}{\partial K_{t+1}} = \frac{1}{C_t} (-1) + \beta E_t V(K_{t+1}, Z_{t+1}) \quad (16)$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} Z_{t+1} \rho K_{t+1}^{\rho-1} (\eta^t H_{t+1})^{1-\rho} + (1 - \delta) \right] \quad (17)$$

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[\frac{Y_{t+1}}{K_{t+1}} \rho + (1 - \delta) \right] \right\} \quad (18)$$

B The steady state and log-linearization

B.1 The steady state

The complete model economy:

$$Y_t = C_t + I_t \quad (19)$$

$$Y_t = Z_t K_t^\rho (\eta^t H_t)^{1-\rho} \quad (20)$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (21)$$

$$\gamma = \frac{Y_t}{C_t} (1 - \rho) \frac{1}{H_t} \quad (22)$$

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[\left(\frac{Y_{t+1}}{K_{t+1}} \right)^\rho + (1 - \delta) \right] \right\} \quad (23)$$

$$\log Z_t = (1 - \psi) \log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2) \quad (24)$$

In equilibrium, $y_t = Y_t/\eta^t$, $c_t = C_t/\eta^t$, $i_t = I_t/\eta^t$, $h_t = H_t$, $k_t = K_t/\eta^t$, and $z_t = Z_t$, therefore we can rewrite the model as:

$$y_t = c_t + i_t$$

$$y_t = z_t k_t^\rho h_t^{1-\rho}$$

$$\eta k_{t+1} = (1 - \delta)k_t + i_t$$

$$\gamma = \frac{y_t}{c_t} (1 - \rho) \frac{1}{h_t}$$

$$\frac{\eta}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}} \left[\left(\frac{y_{t+1}}{k_{t+1}} \right)^\rho + (1 - \delta) \right] \right\}$$

$$\log z_t = (1 - \psi) \log \bar{Z} + \psi \log z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$$

In steady state we have $y_t = y$, $c_t = c$, $i_t = i$, $h_t = h$, $k_t = k$, and $z_t = z$ for all $t = 0, 1, 2, \dots$. Solving for the steady state values of the six variables:

$$a = A \quad (25)$$

$$k = \left(\frac{\rho}{\eta/\beta - 1 + \delta} \right) y \quad (26)$$

$$i = \left[\frac{\rho(\eta - 1 + \delta)}{\eta/\beta - 1 + \delta} \right] y \quad (27)$$

$$c = \left\{ 1 - \left[\frac{\rho(\eta - 1 + \delta)}{\eta/\beta - 1 + \delta} \right] \right\} y \quad (28)$$

$$h = \left(\frac{1 - \rho}{\gamma} \right) \left\{ 1 - \left[\frac{\rho(\eta - 1 + \delta)}{\eta/\beta - 1 + \delta} \right] \right\}^{-1} \quad (29)$$

$$y = z^{1/(1-\rho)} \left(\frac{\rho}{\eta/\beta - 1 + \delta} \right)^{\rho/(1-\rho)} \left(\frac{1 - \rho}{\gamma} \right) \left\{ 1 - \left[\frac{\rho(\eta - 1 + \delta)}{\eta/\beta - 1 + \delta} \right] \right\}^{-1} \quad (30)$$

B.2 Log-linearization

This section presents the log-linearized DSGE model. The principle of log-linearization is to replace all equations by Taylor approximation around the steady state, which are linear functions in the log-deviations of the variables (Uhlig, 1995:4). Suppose Π_t be the vector of variables, π their steady state, and $\hat{\pi}_t$ the vector of log-deviations:

$$\hat{\pi}_t = \log \Pi_t - \log \pi \quad (31)$$

in other words, $\hat{\pi}_t$ denote the percentage deviations from their steady state levels. Using first-order Taylor approximations to rewrite all the equations of the model:

$$\hat{y}_t = \hat{z}_t + \rho \hat{k}_t + (1 - \rho) \hat{h}_t \quad (32)$$

$$\hat{z}_t = \psi \hat{z}_{t-1} + \varepsilon_t \quad (33)$$

$$\left(\frac{\eta}{\beta} - 1 + \delta \right) \hat{y}_t = \left[\left(\frac{\eta}{\beta} - 1 + \delta \right) - \rho(\eta - 1 + \delta) \right] \hat{c}_t + \rho(\eta - 1 + \delta) \hat{i}_t \quad (34)$$

$$\eta \hat{k}_{t+1} = (1 - \delta) \hat{k}_t + (\eta - 1 + \delta) \hat{i}_t \quad (35)$$

$$\hat{c}_t + \hat{h}_t = \hat{y}_t \quad (36)$$

$$0 = \frac{\eta}{\beta} \hat{c}_t - \frac{\eta}{\beta} E_t \hat{c}_{t+1} + \left(\frac{\eta}{\beta} - 1 + \delta \right) E_t \hat{y}_{t+1} - \left(\frac{\eta}{\beta} - 1 + \delta \right) \hat{k}_{t+1} \quad (37)$$

C Forecast accuracy

Table 1. RMSE (2001Q1-2005Q4): Output

QA	1	2	3	4	AVE
DSGE-VAR (Uncon)	<i>1.2432</i>	<i>1.7841</i>	<i>1.8214</i>	<i>1.7765</i>	<i>1.6563</i>
DSGE-VAR (Con)	1.3671	2.0595	2.4382	2.8072	2.1680
VAR (1)	1.4611	2.3092	2.8747	3.4087	2.5134
BVAR (w=.3, d=.5)	0.6698	1.0454	1.3164	1.5712	1.1507

QA: quarter ahead; RMSE: root mean squared error.

Table 2. RMSE (2001Q1-2005Q4): Consumption

QA	1	2	3	4	AVE
DSGE-VAR (Uncon)	<i>1.2001</i>	<i>1.7884</i>	<i>2.1229</i>	<i>2.2967</i>	<i>1.8520</i>
DSGE-VAR (Con)	1.2287	1.9548	2.5158	3.0207	2.1800
VAR (1)	1.2029	1.7833	2.181	2.4643	1.9079
BVAR (w=.3, d=.5)	0.5215	0.7080	0.8293	0.8570	0.7290

QA: quarter ahead; RMSE: root mean squared error.

Table 3. RMSE (2001Q1-2005Q4): Investment

QA	1	2	3	4	AVE
DSGE-VAR (Uncon)	<i>2.8404</i>	<i>3.5985</i>	4.1179	4.1522	3.6773
DSGE-VAR (Con)	2.9518	3.6293	<i>3.9484</i>	<i>4.0228</i>	<i>3.6381</i>
VAR (1)	3.0437	4.3241	5.5072	6.4486	4.8309
BVAR (w=.3, d=.5)	1.1230	1.4757	1.8097	2.0608	1.6173

QA: quarter ahead; RMSE: root mean squared error.

Table 4. RMSE (2001Q1-2005Q4): Hours worked

QA	1	2	3	4	AVE
DSGE-VAR (Uncon)	2.5066	3.3475	4.0577	4.5857	3.6244
DSGE-VAR (Con)	<i>2.4477</i>	<i>2.9966</i>	<i>3.5075</i>	<i>3.7018</i>	<i>3.1634</i>
VAR (1)	2.3913	2.941	3.2884	3.2920	2.9782
BVAR5 (w=.3, d=.5)	1.2420	1.6435	1.8927	1.9342	1.6781

QA: quarter ahead; RMSE: root mean squared error.